Serie 08 - Solution

Preamble

1.1 Sign convention

Here we want to quickly talk about the difference between "difference" and "drop". Let's take an example.

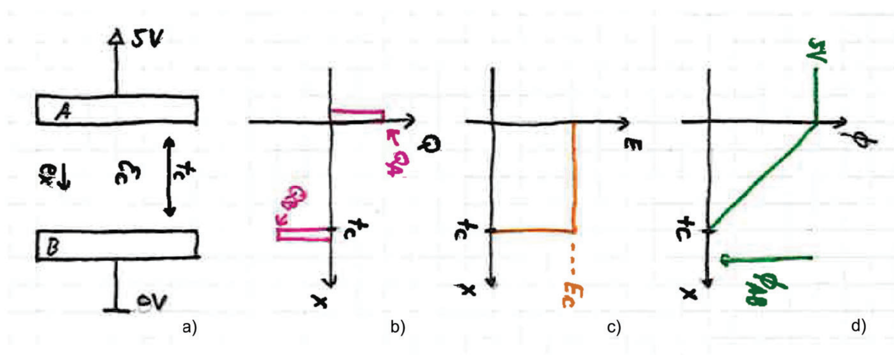

Figure 1: a) Capacitor topology. b) Charge distribution. c) Electric field distribution. d) Electric potential distribution.

Take a capacitor with plate A connected to a voltage of $5\,[V]$, and its plate B connected to ground. The two plates are separated by a distance t_c by an insulator with a dielectric constant ϵ_c. As electrical students, you will all agree that the voltage difference between plates A and B will be $V_{AB} = 5\,[V]$. Now, we want to find this by integrating Maxwell's equations. In the capacitor, the electric field will be:

$$\vec{E_c} = \frac{Q_A}{\epsilon_c}\vec{e_x} \tag{1}$$

We now want to find the electric potential, and its physical definition is:

$$\phi = -\int \vec{E}d\vec{l} \tag{2}$$

If we integrate this over the capacitor:

$$\phi_{AB} = -\int_A^B E_c dx = -\int_0^{t_c} E_c dx = -\frac{Q_A}{\epsilon_c}\cdot x\Big|_{x=t_c}^{0} = -\frac{Q_A}{\epsilon_c}t_c \tag{3}$$

Since Q_A is positive, ϕ_{AB} is negative. This is because this is the calculation for the potential difference, which is, by definition, the inverse of the potential drop.

Because of this, knowing which sign to use can be a nightmare. In this exercise series, we will use ϕ (the only exception is the work function because of its definition) when we talk about differences, and V when we use drop. At the exam, you don't have to use this definition; the course doesn't use it either. We just use this to try to make the sign easier to find.

Here is a small table of correspondences between the course and this solution:

exercises	$\Rightarrow$	course
-ϕ_{ox}	$\Rightarrow$	V_{ox}
-ϕ_{sc}	$\Rightarrow$	V_b
-ϕ_b	$\Rightarrow$	ϕ_b

Given constants

$kT/q = 25.9\,[mV] \quad @ \quad T = 300\,[K]$
$n_i(Si) = 1.5 \cdot 10^{10}\,[cm^{-3}] \quad @ \quad T = 300\,[K]$
$q = 1.60 \cdot 10^{-19}\,[C]$
$\epsilon_0 = 8.85 \cdot 10^{-14}\,[F/cm]$
$\epsilon_{Si} = 11.7 \cdot \epsilon_0$
$\epsilon_{SiO} = 3.9 \cdot \epsilon_0$

Exercise 01

Find the threshold voltage V_T of a MOS structure composed of an aluminum gate, a silicon oxide layer with a thickness of $t_{ox} = 50\,[nm]$, and a silicon substrate doped at $N_a = 10^{14}\,\left[cm^{-3}\right]$. The structure contains interfacial charges with a density of $Q_{ss} = 10^{10}\,\left[cm^{-2}\right]$. Assume a work function difference of $\phi_{ms} = -0.83\,[V]$ and a temperature of $T = 300\,[K]$.

Solution

As drawn in the Fig. 2 the total voltage drop on the MOS structure $\phi_b = \phi_{ms} - V_{gb}$ is distributed among the oxide layer ϕ_{ox} and the semiconductor base ϕ_{sc}.

$$\phi_b = \phi_{ms} - V_{gb} = \phi_{ox} + \phi_{sc} \tag{4}$$

We know that the threshold voltage is define when the potential build in the SCR of the base reach $\phi_{sc} = 2\phi_p$. Therefore we have to find a way to extract the potential difference on the oxide layer. We know the definition of the potential as:

$$\phi_{AB} = -\int_A^B \vec{E} \cdot d\vec{l} \Rightarrow \phi_{ox} = -E_{ox} t_{ox} \tag{5}$$

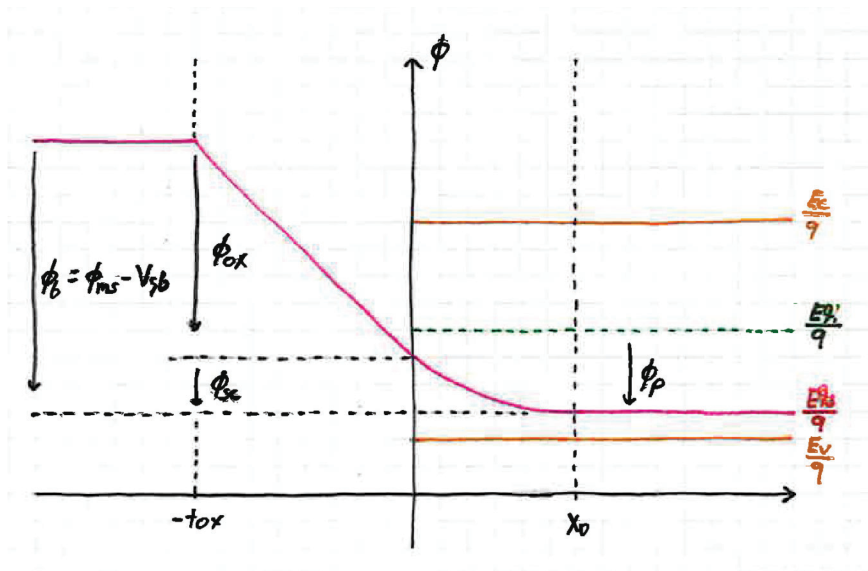

Figure 2: Drawing of the electric potential in the MOS structure.

We already known that the electric field $\vec{E}$ is constant thought the oxide layer. By the rewriting of the Maxwell's equations in our case:

$$E = \frac{Q_m}{\epsilon_{ox}} \tag{6}$$

Where Q_m is the charge accumulated on the gate, and $\epsilon_{ox} = \epsilon_{SiO}$. For this reason we need to know how many charges are store on the gate, fortunately we know that the MOS structure should be neutral.

$$Q_m + qQ_{ss} - qN_aX_D = 0 \Rightarrow Q_m = qN_aX_D - qQ_{ss} \tag{7}$$

Be careful in this exercise Q_{ss} is the interfacial charge density, to have the charge we have to multiply it by the elementary charge. We finally have to find the depletion width at threshold voltage. Fortunately the build in potential on the base only depend on the depletion width (in the case that we are between the flat-band and the threshold), and the dependency is given by the formula seen in coures:

$$\phi_{sc} = -\frac{qN_aX_D^2}{2\epsilon_{sc}} \tag{8}$$

In our case we are at threshold therefore ϕ_{sc} also call V_B is equal to $2\phi_p$ with:

$$\phi_p = -\frac{kT}{q}\ln\left(\frac{N_a}{n_i}\right) \approx -228\,[mV] \tag{9}$$

In our case $\epsilon_{sc} = \epsilon si$, we just have to extract X_D from Eq. 8:

$$X_{Dth} = \sqrt{-\frac{2 \cdot (2\phi_p) \cdot \epsilon_{sc}}{qN_a}} \approx 2.43 \, [um] \tag{10}$$

We can now calculate the charge in the base at threshold Q_{scth}:

$$Q_{scth} = -qN_a X_{Dth} \approx -3.89 \left[\frac{nC}{cm^2} \right] \tag{11}$$

We will now redefine Eq. 7 about charge neutrality with Q_{scth}. We inject the result in the definition of the electric field Eq. 6 and after in electric potential definition Eq. 5.

$$\phi_{ox} = (Q_{scth} + qQ_{ss}) \frac{t_{ox}}{\epsilon_{ox}} \approx -33.1 \, [mV] \tag{12}$$

We now rewrite the Eq. 4 and in our condition the gate base V_{gb} voltage is the threshold voltage therefore $V_{gb} = V_{th}$:

$$\phi_{ms} - V_{th} = (Q_{scth} + qQ_{ss}) \frac{t_{ox}}{\epsilon_{ox}} + 2\phi_p \approx -489 \, [mV] \tag{13}$$

And Finlay we just extract the gate-base bias voltage V_{gb}.

$$V_{th} = \phi_{ms} - (Q_{scth} + qQ_{ss}) \frac{t_{ox}}{\epsilon_{ox}} - 2\phi_p \approx -341 \, [mV] \tag{14}$$

To Go Further

In this section, we will connect this solution with the formula seen during the course. Firstly, we will substitute the final equation Eq. 14 into the definition of Q_{scth} Eq. 11 and the definition of X_{Dth} Eq. 10. The result is as follows:

$$V_{th} = \phi_{ms} - \left(-qN_a \sqrt{-\frac{2 \cdot (2\phi_p) \cdot \epsilon_{sc}}{qN_a}} + qQ_{ss} \right) \frac{t_{ox}}{\epsilon_{ox}} - 2\phi_p \tag{15}$$

By using the definition of the oxide capacitor:

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} \tag{16}$$

we can rewrite this equation as:

$$V_{th} = \phi_{ms} - qQ_{ss} \frac{1}{C_{ox}} - 2\phi_p + \frac{1}{C_{ox}} \sqrt{2\epsilon_{sc} q N_a} \cdot \sqrt{-2\phi_p} \tag{17}$$

By taking a closer look, we can see that the body factor γ has been defined during the course:

$$\gamma = \frac{1}{C_{ox}} \sqrt{2\epsilon_{sc} q N_a} \tag{18}$$

In **Exercise 03** of **Serie 07**, we have seen that the flat-band voltage in the case of interfacial charges can be written as:

$$V_{FB} = \phi_{ms} - \frac{qQ_{ss}}{C_{ox}} \tag{19}$$

Therefore, we can refine the equation given in the course:

$$V_{th} = V_{FB} - 2\phi_p + \gamma\sqrt{-2\phi_p} \tag{20}$$

In this case, $\gamma\sqrt{-2\phi_p}$ represents the contribution of the depletion region on the oxide potential difference.

Of course, you can directly use this formula to solve these exercises.

Exercise 02

Find the oxide thickness t_{ox} of a MOS structure to have a threshold voltage $V_T = 0.65\,[V]$. The MOS structure is composed of an aluminum gate, a silicon oxide layer, and a silicon substrate doped at $N_a = 3 \cdot 10^{16}\left[cm^{-3}\right]$. The structure contains interfacial charge with a density of $Q_{SS} = 10^{11}\left[cm^{-2}\right]$. Assume a work function difference of $\phi_{ms} = -1.13\,[V]$ and a temperature of $T = 300\,[K]$.

Solution

An intuitive reasoning can be applied to this exercise to extract the oxide thickness. As this reasoning will be quite similar to the solution of the precedent one, it will not been given in detail here. But if you are not comfortable with MOS structure we would only encourage you to do so. we just give you the main step.

1. Find ϕ_{ox} we already have V_{gb}, ϕ_{ms} and $\phi_{sc} = 2\phi_p$.
2. Find Q_m by MOS charge neutrality we can calculate Q_{scth} and we have Q_{ss}.
3. Find E_{ox}.
4. Find t_{ox} with E_{ox} and ϕ_{ox}.

In our case we will just take the solution of the precedent exercise Eq. 14 and extract t_{ox}.

$$\phi_p = -\frac{kT}{q}\ln\left(\frac{N_a}{n_i}\right) \approx -376\,[mV] \tag{21}$$

$$X_{Dth} = \sqrt{-\frac{2 \cdot (2\phi_p) \cdot \epsilon_{sc}}{qN_a}} \approx 180\,[nm] \tag{22}$$

$$Q_{scth} = -qN_aX_{Dth} \approx -86.4 \left[\frac{nC}{cm^2}\right] \tag{23}$$

And finally:

$$t_{ox} = \frac{\phi_{ms} - V_{gb} - 2\phi_p}{Q_{scth} + qQ_{ss}}\epsilon_{ox} \approx 50.4\,[nm] \tag{24}$$

Exercise 03

Calculate C_{ox} and the minimum capacitance C_{min} of a MOS structure composed of an aluminum gate, a silicon oxide layer with a thickness of $t_{ox} = 55\,[nm]$, and a silicon substrate doped at $N_a = 10^{16}\,[cm^{-3}]$.

Solution

As we have seen in previous series, the depletion region increases until we reach the threshold voltage, after which an inversion layer is created at the interface between the oxide and the channel. A capacitor is basically two conductive parts separated by an insulating part. The evolution of the gate capacitor depends on the depletion region and follows analogous reasoning to that seen in the junction capacitor of a diode.

1. Before the flat band, there is no space-charge region (SCR), therefore the gate capacitor is equal to C_{ox}.
2. Between the flat band and threshold, there is an SCR, which acts as an insulator and increases the insulator distance. Therefore, the capacitance will drop.
3. After the threshold, the inversion layer is localized close to the oxide, but it takes time to be created. Therefore, for low frequency, the capacitor will return to C_{ox} and at high frequency, it will remain the same as at the threshold.

$$C_{max} = C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} \approx 62.8 \left[\frac{nF}{cm^2}\right] \tag{25}$$

Now, we will find the depletion width at the threshold in the same way that we have done in the two previous exercises.

$$\phi_p = -\frac{kT}{q}\ln\left(\frac{N_a}{n_i}\right) \approx -347\,[mV] \tag{26}$$

$$X_{Dth} = \sqrt{-\frac{2 \cdot (2\phi_p) \cdot \epsilon_{sc}}{qN_a}} \approx 300\,[nm] \tag{27}$$

Now, we have a capacitor composed of two different permittivities. One way to find the total capacitance is to consider that it is two capacitors in series. In this case, the capacitor of the SCR is:

$$C_{sc} = \frac{\epsilon_{sc}}{X_{Dth}} \approx 34.5 \left[\frac{nF}{cm^2}\right] \tag{28}$$

The capacitance value of two capacitors in series is:

$$C_{min} = \frac{1}{\frac{1}{C_{ox}} + \frac{1}{C_{sc}}} \approx 22.3 \left[\frac{nF}{cm^2}\right] \tag{29}$$